

Scalar Mass Bounds in Two Supersymmetric Extended Electroweak Gauge Models

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Abstract

In two recently proposed supersymmetric extended electroweak gauge models, the reduced Higgs sector at the 100-GeV energy scale consists of only two doublets, but they have quartic scalar couplings different from those of the minimal supersymmetric standard model. In the $SU(2) \times SU(2) \times U(1)$ model, there is an absolute upper bound of about 145 GeV on the mass of the lightest neutral scalar boson. In the $SU(3) \times U(1)$ model, there is only a parameter-dependent upper bound which formally goes to infinity in a particular limit.

It has been demonstrated recently in two explicit examples[1, 2] that in a supersymmetric extension of the standard $SU(2)_L \times U(1)_Y$ electroweak gauge model, even though there may be only two scalar doublets at the electroweak energy scale, they are not necessarily those of the minimal supersymmetric standard model (MSSM). The reason is that the $SU(2) \times U(1)$ gauge symmetry may be a remnant[3] of a larger symmetry which is broken at a higher mass scale together with the supersymmetry. The structure of the reduced Higgs potential is then determined by the scalar particle content of the larger theory and there are in general additional couplings appearing in the superpotential which contribute to the quartic couplings of the two Higgs doublets at the electroweak energy scale.

Consider the Higgs potential V for two scalar doublets $\Phi_{1,2} = (\phi_{1,2}^+, \phi_{1,2}^0)$:

$$\begin{aligned} V = & \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \mu_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) \\ & + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) \\ & + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \frac{1}{2} \lambda_5^* (\Phi_2^\dagger \Phi_1)^2. \end{aligned} \quad (1)$$

In the MSSM, there are the well-known constraints

$$\lambda_1 = \lambda_2 = \frac{1}{4}(g_1^2 + g_2^2), \quad \lambda_3 = -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2, \quad \lambda_4 = -\frac{1}{2}g_2^2, \quad \lambda_5 = 0, \quad (2)$$

where g_1 and g_2 are the $U(1)$ and $SU(2)$ gauge couplings of the standard model respectively. Note that only the gauge couplings contribute to the λ 's. This is because that with two $SU(2) \times U(1)$ Higgs superfields, there is no cubic invariant in the superpotential and thus no additional coupling.

In the supersymmetric $SU(2)_L \times SU(2)_R \times U(1)$ model based on E_6 particle content,[4] it has been shown[1] that instead of Eq. (2), the quartic scalar couplings of the two Higgs doublets are now given by

$$\lambda_1 = \frac{1}{4}(g_1^2 + g_2^2) + \frac{2f^2 g_1^2}{g_2^2} - \frac{4f^4}{g_2^2} \left(1 - \frac{g_1^2}{g_2^2}\right), \quad (3)$$

$$\lambda_2 = \frac{1}{4}(g_1^2 + g_2^2) + 2f^2 \left(1 - \frac{g_1^2}{g_2^2}\right) - \frac{4f^4}{g_2^2} \left(1 - \frac{g_1^2}{g_2^2}\right), \quad (4)$$

$$\lambda_3 = -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2 + f^2 - \frac{4f^4}{g_2^2} \left(1 - \frac{g_1^2}{g_2^2}\right), \quad (5)$$

$$\lambda_4 = -\frac{1}{2}g_2^2 + f^2, \quad \lambda_5 = 0, \quad (6)$$

where f is a coupling in the superpotential of the larger theory.[1] The general requirement that V be bounded from below puts an upper limit on f : namely

$$f_{max}^2 = \frac{1}{4}(g_1^2 + g_2^2) \left(1 - \frac{g_1^2}{g_2^2}\right)^{-1}, \quad (7)$$

which corresponds to having soft supersymmetry breaking terms in the Higgs sector which also preserve the left-right symmetry. If $f = f_{max}$, the lighter of the two neutral Higgs bosons is constrained by[1]

$$m_h^2 < 2M_W^2 \left[1 - \frac{2x_W^2}{(1 - x_W)(1 - 2x_W)}\right] \sin^4 \beta + \epsilon, \quad (8)$$

where

$$\epsilon = \frac{3g_2^2 m_t^4}{8\pi^2 M_W^2} \ln \left(1 + \frac{\tilde{m}^2}{m_t^2}\right) \quad (9)$$

is an extra term coming from radiative corrections[5] and $x_W \equiv \sin^2 \theta_W$, $\tan \beta \equiv \langle \phi_2^0 \rangle / \langle \phi_1^0 \rangle$. Using $m_t = 174$ GeV[6] and assuming $\tilde{m} = 1$ TeV, this gives an upper bound of about 133 GeV on m_h instead of about 128 GeV if the MSSM were assumed. However, the choice of $f = f_{max}$ does not maximize the upper bound on m_h , as shown below.

For a Higgs potential with $\lambda_5 = 0$, the upper bound on the lighter of the two neutral scalar Higgs bosons is given in general by

$$(m_h^2)_{max} = 2(v_1^2 + v_2^2)[\lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + 2(\lambda_3 + \lambda_4) \sin^2 \beta \cos^2 \beta] + \epsilon, \quad (10)$$

where $v_{1,2} = \langle \phi_{1,2}^0 \rangle$. Using Eqs. (3) to (6), we then obtain

$$(m_h^2)_{max} = \frac{1}{2}(v_1^2 + v_2^2)[(g_1^2 + g_2^2) \cos^2 2\beta + 8f^2(1 - \cos^4 \beta + (g_1^2/g_2^2) \cos 2\beta) - 16(f^4/g_2^2)(1 - g_1^2/g_2^2)] + \epsilon, \quad (11)$$

which is maximized at

$$f_0^2 = \frac{1}{4}g_2^2 \left(1 - \cos^4 \beta + \frac{g_1^2}{g_2^2} \cos 2\beta \right) \left(1 - \frac{g_1^2}{g_2^2} \right)^{-1}. \quad (12)$$

Since f_0 never exceeds f_{max} , the former should be used to maximize $(m_h^2)_{max}$ and we obtain

$$(m_h^2)_{max} < 2(v_1^2 + v_2^2) \left[\frac{1}{4}(g_1^2 + g_2^2) \cos^2 2\beta + \frac{g_2^2(1 - \cos^4 \beta + (g_1^2/g_2^2) \cos 2\beta)^2}{4(1 - g_1^2/g_2^2)} \right] + \epsilon. \quad (13)$$

We plot this upper bound (solid line) as a function of $\cos^2 \beta$ in Fig. 1. The absolute upper bound is reached at $\cos^2 \beta = 0$ where it is equal to $(2M_W^2 + \epsilon)^{1/2}$ which is about 145 GeV. For comparison, we show the corresponding upper bound (dashed line) for $f = f_{max}$. We show also the upper bound (dotted line) of the MSSM, namely $(M_Z^2 \cos^2 2\beta + \epsilon)^{1/2}$, corresponding to $f = 0$.

In the supersymmetric extension of the new $SU(3) \times U(1)$ model[7] the reduced Higgs potential of two scalar doublets at the electroweak energy scale is obtained[2] with the following quartic scalar couplings:

$$\lambda_1 = \frac{1}{4}(g_1^2 + g_2^2) + \left(1 + \frac{3g_1^2}{g_2^2} \right) \frac{x}{1-y} - \frac{3}{g_2^2} \left(1 - \frac{3g_1^2}{g_2^2} \right) \frac{(1+yr)x^2}{(1-y)^2}, \quad (14)$$

$$\lambda_2 = \frac{1}{4}(g_1^2 + g_2^2) + \left(1 - \frac{3g_1^2}{g_2^2} \right) \frac{x}{1-y} - \frac{3}{g_2^2} \left(1 - \frac{3g_1^2}{g_2^2} \right) \frac{(1+yr)x^2}{(1-y)^2}, \quad (15)$$

$$\lambda_3 = -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2 + \frac{x}{1-y} - \frac{3}{g_2^2} \left(1 - \frac{3g_1^2}{g_2^2} \right) \frac{(1+yr)x^2}{(1-y)^2}, \quad (16)$$

$$\lambda_4 = -\frac{1}{2}g_2^2 + x, \quad \lambda_5 = 0, \quad (17)$$

where $x \equiv f^2$, $y \equiv (u'/u)^2$, and $r \equiv (M/M')^2$. Here, f is the extra coupling appearing in the superpotential of the larger theory, u and u' are the vacuum expectation values of the two $SU(3) \times U(1)$ scalar triplets which break the gauge symmetry down to the standard $SU(2) \times U(1)$, M' is the arbitrary mass of the corresponding heavy pseudoscalar boson, and

$$M^2 = \frac{2}{3}g_2^2 \left(1 - \frac{3g_1^2}{g_2^2} \right)^{-1} (u^2 + u'^2). \quad (18)$$

Details have been given elsewhere.[2] Using Eq. (10) we then find

$$(m_h^2)_{max} = \frac{1}{2}(v_1^2 + v_2^2)[(g_1^2 + g_2^2) \cos^2 2\beta + 4x(1-y)^{-1}\{1 + 3(g_1^2/g_2^2) \cos 2\beta + \frac{1}{2} \sin^2 2\beta(1-y)\} - 12x^2 g_2^{-2}(1 - 3g_1^2/g_2^2)(1+yr)(1-y)^{-2}] + \epsilon. \quad (19)$$

In the above, r appears only in one term and since that term is necessarily negative, $(m_h^2)_{max}$ is always maximized with the smallest possible value of r . Hence we need only consider $r = 0$ from now on.

The requirement that V be bounded from below implies that $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, and either $\lambda_3 + \lambda_4 \geq 0$ or if $\lambda_3 + \lambda_4 < 0$, then $|\lambda_3 + \lambda_4| \leq \sqrt{\lambda_1 \lambda_2}$. We show in Fig. 2 the resulting upper bound on x as a function of y . There are four separate regions:

$$0 \leq y \leq 1, \quad \text{then} \quad x \leq x_2 = \frac{1}{6}g_2^2(1-y) \left[1 + \frac{2}{(1-3g_1^2/g_2^2)^{1/2}} \right], \quad (20)$$

$$1 \leq y \leq 3, \quad \text{then} \quad x = 0, \quad (21)$$

$$3 \leq y \leq 4, \quad \text{then} \quad x \leq x_3, \quad (22)$$

$$y \geq 4, \quad \text{then} \quad x \leq x_1 = \frac{1}{6}g_2^2(y-1). \quad (23)$$

In the above, x_1 and x_2 correspond to $\lambda_1 = 0$ and $\lambda_2 = 0$ respectively, whereas x_3 is given by the smaller of the two roots of the quadratic equation $[\lambda_1 \lambda_2 - (\lambda_3 + \lambda_4)^2]/x = 0$, namely

$$\frac{3}{g_2^2} \left(1 - \frac{3g_1^2}{g_2^2} \right) x^2 - \left[1 - \frac{3g_1^2}{g_2^2} + \frac{1}{2}(y-2)^2 \right] x + \frac{1}{4}(g_1^2 + g_2^2)(y-1)(y-3) = 0. \quad (24)$$

Previously[2] we had considered only the special case $y = 0$ and further assumed that $(\lambda_3 + \lambda_4)^2 \leq \lambda_1 \lambda_2$ so that the pseudoscalar mass m_A is always allowed to be zero. If $\lambda_3 + \lambda_4 \geq \sqrt{\lambda_1 \lambda_2}$, then m_A has a lower bound given by

$$m_A^2 \geq \frac{4[(\lambda_3 + \lambda_4)^2 - \lambda_1 \lambda_2]v_1^2 v_2^2}{(m_h^2)_{max}} \quad (25)$$

which is in principle not a problem.

For a given value of y , we now vary x to maximize the value of $(m_h^2)_{max}$. Let $z \equiv \cos^2 \beta$, then

$$x_0 = \frac{1}{6}g_2^2(1-y) \left(1 - \frac{3g_1^2}{g_2^2}\right)^{-1} \left[1 + \frac{3g_1^2}{g_2^2}(2z-1) + 2z(1-z)(1-y)\right] \quad (26)$$

would be the value of x at which $(m_h^2)_{max}$ is maximized for $0 \leq y < 1$, unless $x_0 > x_2$ in which case x_2 should be used instead. For a given value of z , it can be shown that $(m_h)_{max}$ is maximized at $y = 0$ with x_0 for $z < 0.176$ and x_2 for $z > 0.176$. We plot in Fig. 3 this upper bound as a function of z , as well as lines for some other values of y . For any value in the range $0 \leq y < 1$, $(m_h^2)_{max}$ is maximized at $z = 1$ where it is equal to

$$2g_1^2(v_1^2 + v_2^2) \left[1 + \frac{2}{(1 - 3g_1^2/g_2^2)^{1/2}}\right] + \epsilon. \quad (27)$$

Hence m_h has an absolute upper bound of about 258 GeV in this interval. However, it should be noted that near $y = 1$, our approximation for the reduced Higgs potential breaks down because one of the mass eigenvalues of the heavy neutral scalar mass matrix becomes too small. On the other hand, this problem does not affect the upper bound on $(m_h)_{max}$ for any given value of z because it always occurs at $y = 0$ as shown in Fig. 3.

In the interval $1 \leq y \leq 3$, since $x = 0$ is required, this model is identical to the MSSM in its reduced scalar sector. Hence m_h is bounded by the dotted line of Fig. 1.

In the interval $3 \leq y \leq 4$, we need to consider the lines $x_0 = 0$ and $x_0 = x_3$ in the (y, z) plane. For a given value of y , $(m_h)_{max}$ is maximized with $x = 0$ for $z < z_1$ and $z > z_2$, where $z_{1,2}$ are the roots of the quadratic equation

$$z^2 - z \left(1 - \frac{3g_1^2/g_2^2}{y-1}\right) + \frac{1 - 3g_1^2/g_2^2}{2(y-1)} = 0, \quad (28)$$

with $x = x_0$ for $z_1 < z < z_3$ and $z_4 < z < z_2$, and with $x = x_3$ for $z_3 < z < z_4$, where $z_{3,4}$ are the roots of the quadratic equation

$$z^2 - z \left(1 - \frac{3g_1^2/g_2^2}{y-1}\right) + \frac{1 - 3g_1^2/g_2^2}{2(y-1)} \left(1 + \frac{6x_3/g_2^2}{y-1}\right) = 0. \quad (29)$$

We plot in Fig. 4 this upper bound as a function of z for various values of y .

For $y \geq 4$, we need to consider the lines $x_0 = 0$ and $x_0 = x_1$. For a given value of y , $(m_h)_{max}$ is maximized with $x = 0$ again for $z < z_1$ and $z > z_2$, with $x = x_0$ now for $z_1 < z < z_5$ and $z_6 < z < z_2$, and with $x = x_1$ for $z_5 < z < z_6$, where $z_{5,6}$ are the roots of the quadratic equation

$$z^2 - z \left(1 - \frac{3g_1^2/g_2^2}{y-1} \right) + \frac{1 - 3g_1^2/g_2^2}{y-1} = 0. \quad (30)$$

We plot in Fig. 5 this upper bound on $(m_h)_{max}$ as a function of z for various values of y .

We find that for $y - 4 > 6g_1^2/g_2^2$, $(m_h^2)_{max}$ at $x = x_1$ has a local maximum at

$$z = \frac{1}{2} \left[1 - \frac{3g_1^2/g_2^2}{y-4-3g_1^2/g_2^2} \right], \quad (31)$$

where it is equal to

$$\frac{1}{6} g_2^2 (v_1^2 + v_2^2) \frac{(y-4)^2}{y-4-3g_1^2/g_2^2} + \epsilon. \quad (32)$$

It is clear that this expression is unbounded as y increases and it exceeds $M_Z^2 + \epsilon$ at $y \simeq 6.5$, and the upper bound (27) at $y \simeq 30.5$. However, as Eq. (23) shows, this requires the Yukawa coupling f to be increasingly large.

In summary, we plot in Fig. 6 the upper bound on m_h as a function of y . In the interval $0 \leq y < 1$, it is about 258 GeV, then it has the MSSM value of 128 GeV from $y = 1$ to $y \simeq 6.5$, after which it grows according to Eq. (32). Details are shown in Figs. 2 to 5.

The experimental search for Higgs bosons is an active ongoing effort.[8] It should be understood that even if supersymmetry exists and there are only two Higgs doublets at the electroweak energy scale, they are not necessarily those of the MSSM. In particular, the upper bound on m_h may exceed that predicted by the MSSM, as shown here in two specific examples.

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Figure Captions

Fig. 1 Upper bound (solid line) on m_h as a function of $\cos^2 \beta$ in the supersymmetric $SU(2)_L \times SU(2)_R \times U(1)$ model based on E_6 particle content. The dashed and dotted lines refer to the special cases where $f = f_{max}$ and $f = 0$ respectively.

Fig. 2 Upper bound on $x \equiv f^2$ as a function of $y \equiv (u'/u)^2$ in the new supersymmetric $SU(3) \times U(1)$ model.

Fig. 3 Upper bound on m_h as a function of $z \equiv \cos^2 \beta$ in the interval $0 \leq y \leq 1$ for various values of y .

Fig. 4 Upper bound on m_h as a function of z in the interval $3 \leq y \leq 4$ for various values of y .

Fig. 5 Upper bound on m_h as a function of z for various values of $y > 4$.

Fig. 6 Upper bound on m_h as a function of y .

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